Transportation Models

MODULE C

MODULE OUTLINE

Alaska Airlines

Alaska Airlines

LEARNING OBJECTIVES

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LO C.2 ***Solve*** a problem with the stepping-stone method 734

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The problem facing rental companies like Avis, Hertz, and National is cross-country travel. Lots of it. Cars rented in New York end up in Chicago, cars from L.A. come to Philadelphia, and cars from Boston come to Miami. The scene is repeated in over 100 cities around the U.S. As a result, there are too many cars in some cities and too few in others. Operations managers have to decide how many of these rentals should be trucked (by costly auto carriers) from each city with excess capacity to each city that needs more rentals. The process requires quick action for the most economical routing, so rental car companies turn to transportation modeling.

Transportation Modeling

Because location of a new factory, warehouse, or distribution center is a strategic issue with substantial cost implications, most companies consider and evaluate several locations. With a wide variety of objective and subjective factors to be considered, rational decisions are aided by a number of techniques. One of those techniques is transportation modeling.

The transportation models described in this module prove useful when considering alternative facility locations *within the framework of an existing distribution system*. Each new potential plant, warehouse, or distribution center will require a different allocation of shipments, depending on its own production and shipping costs and the costs of each existing facility. The choice of a new location depends on which will yield the minimum cost *for the entire system*.

Transportation modeling
An iterative procedure for solving problems that involves minimizing the cost of shipping products from a series of sources to a series of destinations.

Transportation modeling finds the least-cost means of shipping supplies from several origins to several destinations. *Origin points* (or *sources*) can be factories, warehouses, car rental agencies like Avis, or any other points from which goods are shipped. *Destinations* are any points that receive goods. To use the transportation model, we need to know the following:

1. The origin points and the capacity or supply per period at each.
2. The destination points and the demand per period at each.
3. The cost of shipping one unit from each origin to each destination.

The transportation model is one form of the linear programming models discussed in Business Analytics Module B. Software is available to solve both transportation problems and the more general class of linear programming problems. To fully use such programs, though, you need to understand the assumptions that underlie the model. To illustrate the transportation problem, we now look at a company called Arizona Plumbing, which makes, among other products,

TABLE C.1 Transportation Costs Per Bathtub for Arizona Plumbing

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND
Des Moines	$5	$4	$3
Evansville	$8	$4	$3
Fort Lauderdale	$9	$7	$5

a full line of bathtubs. In our example, the firm must decide which of its factories should supply which of its warehouses. Relevant data for Arizona Plumbing are presented in Table C.1 and Figure C.1. Table C.1 shows, for example, that it costs Arizona Plumbing $5 to ship one bathtub from its Des Moines factory to its Albuquerque warehouse, $4 to Boston, and $3 to Cleveland.

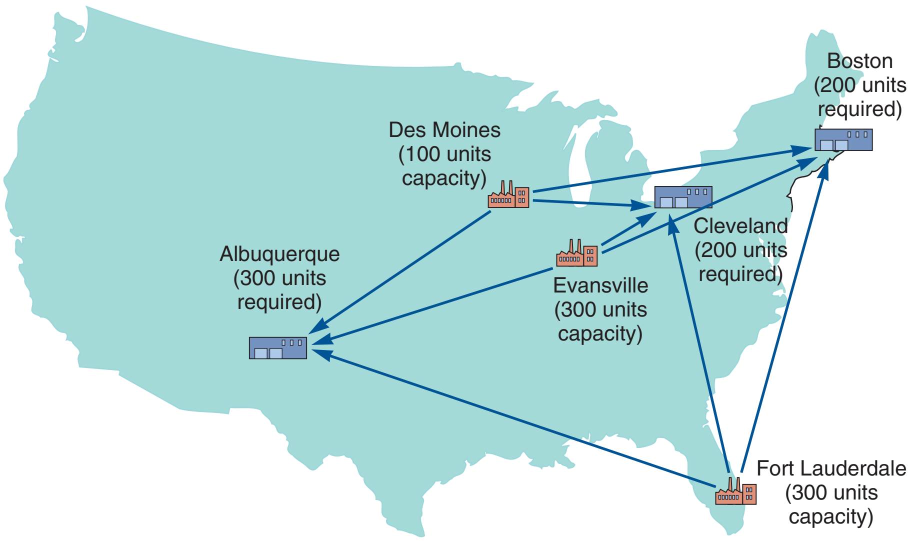


Figure **C.1**

Transportation Problem

Likewise, we see in Figure C.1 that the 300 units required by Arizona Plumbing's Albuquerque warehouse may be shipped in various combinations from its Des Moines, Evansville, and Fort Lauderdale factories.

The first step in the modeling process is to set up a *transportation matrix*. Its purpose is to summarize all relevant data and to keep track of algorithm computations. Using the information displayed in Figure C.1 and Table C.1, we can construct a transportation matrix as shown in Figure C.2.

From \ To	Albuquerque	Boston	Cleveland	Factory capacity
Des Moines	$5	$4	$3	100
Evansville	$8	$4	$3	300
Fort Lauderdale	$9	$7	$5	300
Warehouse requirement	300	200	200	700

Des Moines capacity constraint

Cell representing a possible source-to-destination shipping assignment (Evansville to Cleveland)

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total demand and total supply

Figure **C.2**

Transportation Matrix for Arizona Plumbing

LO C.1 *Develop* an initial solution to a transportation model with the northwest-corner and intuitive lowest-cost methods

Developing an Initial Solution

Once the data are arranged in tabular form, we must establish an initial feasible solution to the problem. A number of different methods have been developed for this step. We now discuss two of them, the northwest-corner rule and the intuitive lowest-cost method.

The Northwest-Corner Rule

Northwest-corner rule
A procedure in the transportation model where one starts at the upper-left-hand cell of a table (the northwest corner) and systematically allocates units to shipping routes.

The **northwest-corner rule** requires that we start in the upper-left-hand cell (or northwest corner) of the table and allocate units to shipping routes as follows:

1. Exhaust the supply (factory capacity) of each row (e.g., Des Moines: 100) before moving down to the next row.
2. Exhaust the (warehouse) requirements of each column (e.g., Albuquerque: 300) before moving to the next column on the right.
3. Check to ensure that all supplies and demands are met.

Example C1 applies the northwest-corner rule to our Arizona Plumbing problem.

Example C1

THE NORTHWEST-CORNER RULE

Arizona Plumbing wants to use the northwest-corner rule to find an initial solution to its problem.

APPROACH ▶ Follow the three steps listed above. See Figure C.3.

SOLUTION ▶ To make the initial solution, these five assignments are made:

1. Assign 100 tubs from Des Moines to Albuquerque (exhausting Des Moines's supply).
2. Assign 200 tubs from Evansville to Albuquerque (exhausting Albuquerque's demand).
3. Assign 100 tubs from Evansville to Boston (exhausting Evansville's supply).
4. Assign 100 tubs from Fort Lauderdale to Boston (exhausting Boston's demand).
5. Assign 200 tubs from Fort Lauderdale to Cleveland (exhausting Cleveland's demand and Fort Lauderdale's supply).

Figure **C.3**
Northwest-Corner Solution to Arizona Plumbing Problem

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5 100	$4	$3	100
(E) Evansville	$8 200	$4 100	$3	300
(F) Fort Lauderdale	$9	$7 (100)	$5 200	300
Warehouse requirement	300	200	200	700

Means that the firm is shipping 100 bathtubs from Fort Lauderdale to Boston

The total cost of this shipping assignment is $4,200 (see Table C.2).

TABLE C.2 **Computed Shipping Cost**

ROUTE				
FROM	**TO**	**TUBS SHIPPED**	**COST PER UNIT**	**TOTAL COST**
D	A	100	$5	$ 500
E	A	200	8	1,600
E	B	100	4	400
F	B	100	7	700
F	C	200	5	$1,000
				Total: $4,200

INSIGHTS ▶ The solution given is feasible because it satisfies all demand and supply constraints. The northwest-corner rule is easy to use, but it totally ignores costs, and therefore should only be considered as a starting position.

LEARNING EXERCISE ▶ Does the shipping assignment change if the cost from Des Moines to Albuquerque increases from \$5 per unit to \$10 per unit? Does the total cost change? [Answer: The initial assignment is the same, but cost = \$4,700.]

RELATED PROBLEMS ▶ C.1a, C.3a, C.15

The Intuitive Lowest-Cost Method

The **intuitive method** makes initial allocations based on lowest cost. This straightforward approach uses the following steps:

Intuitive method
A cost-based approach to finding an initial solution to a transportation problem.

1. Identify the cell with the lowest cost. Break any ties for the lowest cost arbitrarily.
2. Allocate as many units as possible to that cell without exceeding the supply or demand. Then cross out that row or column (or both) that is exhausted by this assignment.
3. Find the cell with the lowest cost from the remaining (not crossed out) cells.
4. Repeat Steps 2 and 3 until all units have been allocated.

Example C2

THE INTUITIVE LOWEST-COST APPROACH

Arizona Plumbing now wants to apply the intuitive lowest-cost approach.

APPROACH ▶ Apply the 4 steps listed above to the data in Figure C.2.

SOLUTION ▶ When the firm uses the intuitive approach on the data (rather than the northwest-corner rule) for its starting position, it obtains the solution seen in Figure C.4.

The total cost of this approach = \$3(100) + \$3(100) + \$4(200) + \$9(300) = \$4,100.
(D to C) (E to C) (E to B) (F to A)

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5	\$4	\$3 / 100	100
(E) Evansville	\$8	\$4 / 200	\$3 / 100	300
(F) Fort Lauderdale	\$9 / 300	\$7	\$5	300
Warehouse requirement	300	200	200	700

First, cross out top row (D) after entering 100 units in \$3 cell because row D is satisfied.

Second, cross out column C after entering 100 units in this \$3 cell because column C is satisfied.

Third, cross out row E and column B after entering 200 units in this \$4 cell because a total of 300 units satisfies row E and column B.

Finally, enter 300 units in the only remaining cell to complete the allocations.

Figure **C.4**

Intuitive Lowest-Cost Solution to Arizona Plumbing Problem

INSIGHT ▶ This method's name is appropriate as most people find it intuitively correct to include costs when making an initial assignment.

LEARNING EXERCISE ▶ If the cost per unit from Des Moines to Cleveland is not \$3, but rather \$6, does this initial solution change? [Answer: Yes, now D − B = 100, D − C = 0, E − B = 100, E − C = 200, F − A = 300. Others unchanged at zero. Total cost stays the same.]

RELATED PROBLEMS ▶ C.1b, C.2, C.3b

Although the likelihood of a minimum-cost solution *does* improve with the intuitive method, we would have been fortunate if the intuitive solution yielded the minimum cost. In this case, as in the northwest-corner solution, it did not. Because the northwest-corner and the intuitive lowest-cost approaches are meant only to provide us with a starting point, we often will have to employ an additional procedure to reach an *optimal* solution.

The Stepping-Stone Method

Stepping-stone method
An iterative technique for moving from an initial feasible solution to an optimal solution in the transportation method.

LO C.2 *Solve* a problem with the stepping-stone method

The **stepping-stone method** will help us move from an initial feasible solution to an optimal solution. It is used to evaluate the cost effectiveness of shipping goods via transportation routes not currently in the solution. When applying it, we test each unused cell, or square, in the transportation table by asking: What would happen to total shipping costs if one unit of the product (for example, one bathtub) was tentatively shipped on an unused route? We conduct the test as follows:

1. Select any unused square to evaluate.
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used (only horizontal and vertical moves are permissible). You may, however, step over either an empty or an occupied square.
3. Beginning with a plus (+) sign at the unused square, place alternating minus signs and plus signs on each corner square of the closed path just traced.
4. Calculate an improvement index by first adding the unit-cost figures found in each square containing a plus sign and then by subtracting the unit costs in each square containing a minus sign.
5. Repeat Steps 1 through 4 until you have calculated an improvement index for all unused squares. If all indices computed are *greater than or equal to zero*, you have reached an optimal solution. If not, the current solution can be improved further to decrease total shipping costs.

Example C3 illustrates how to use the stepping-stone method to move toward an optimal solution. We begin with the northwest-corner initial solution developed in Example C1.

Example C3

CHECKING UNUSED ROUTES WITH THE STEPPING-STONE METHOD

Arizona Plumbing wants to evaluate unused shipping routes.

APPROACH ▶ Start with Example C1's Figure C.3 and follow the 5 steps listed above. As you can see, the four currently unassigned routes are Des Moines to Boston, Des Moines to Cleveland, Evansville to Cleveland, and Fort Lauderdale to Albuquerque.

SOLUTION ▶ **Steps 1 and 2.** Beginning with the Des Moines–Boston route, trace a closed path *using only currently occupied squares* (see Figure C.5). Place alternating plus and minus signs in the corners of this path. In the upper-left square, for example, we place a minus sign because we have *subtracted* 1 unit from the original 100. Note that we can use only squares currently used for shipping to turn the corners of the route we are tracing. Hence, the path Des Moines–Boston to Des Moines–Albuquerque to Fort Lauderdale–Albuquerque to Fort Lauderdale–Boston to Des Moines–Boston would not be acceptable because the Fort Lauderdale–Albuquerque square is empty. It turns out that *only one closed route exists for each empty square*. Once this one closed path is identified, we can begin assigning plus and minus signs to these squares in the path.

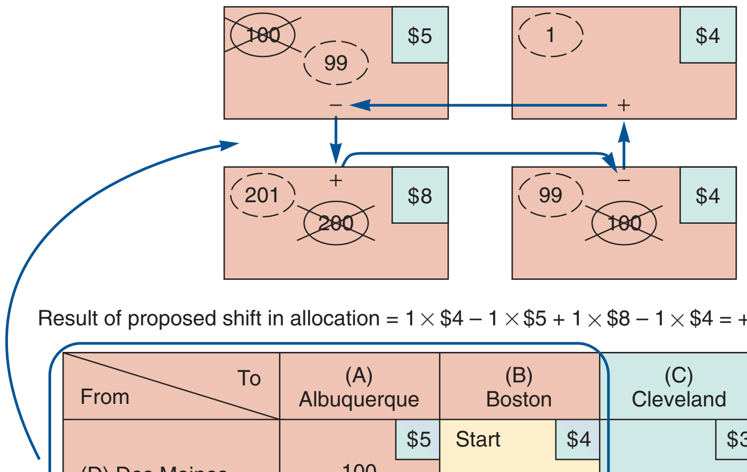
Step 3. How do we decide which squares get plus signs and which squares get minus signs? The answer is simple. Because we are testing the cost-effectiveness of the Des Moines–Boston shipping route, we try shipping 1 bathtub from Des Moines to Boston. This is 1 *more* unit than we *were* sending between the two cities, so place a plus sign in the box. However, if we ship 1 more unit than before from Des Moines to Boston, we end up sending 101 bathtubs out of the Des Moines factory. Because the Des Moines factory's capacity is only 100 units, we must ship 1 bathtub less from Des Moines to Albuquerque. This change prevents us from violating the capacity constraint.

To indicate that we have reduced the Des Moines–Albuquerque shipment, place a minus sign in its box. As you continue along the closed path, notice that we are no longer meeting our Albuquerque warehouse requirement for 300 units. In fact, if we reduce the Des Moines–Albuquerque shipment to 99 units, we must increase the Evansville–Albuquerque load by 1 unit, to 201 bathtubs. Therefore, place a plus sign in that box to indicate the increase. You may also observe that those squares in which we turn a corner (and only those squares) will have plus or minus signs.

Figure C.5

Stepping-Stone Evaluation of Alternative Routes for Arizona Plumbing

Evaluation of Des Moines to Boston square



Result of proposed shift in allocation = 1 × \$4 − 1 × \$5 + 1 × \$8 − 1 × \$4 = + \$3

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5 100 −	\$4 Start +	\$3	100
(E) Evansville	\$8 200 +	\$4 100 −	\$3	300
(F) Fort Lauderdale	\$9	\$7 100	\$5 200	300
Warehouse requirement	300	200	200	700

Finally, note that if we assign 201 bathtubs to the Evansville–Albuquerque route, then we must reduce the Evansville–Boston route by 1 unit, to 99 bathtubs, to maintain the Evansville factory's capacity constraint of 300 units. To account for this reduction, we thus insert a minus sign in the Evansville–Boston box. By so doing, we have balanced supply limitations among all four routes on the closed path.

Step 4. Compute an improvement index for the Des Moines–Boston route by adding unit costs in squares with plus signs and subtracting costs in squares with minus signs.

$$\text{Des Moines–Boston index} = \$4 - \$5 + \$8 - \$4 = +\$3$$

This means that for every bathtub shipped via the Des Moines–Boston route, total transportation costs will increase by \$3 over their current level.

Let us now examine the unused Des Moines–Cleveland route, which is slightly more difficult to trace with a closed path (see Figure C.6). Again, notice that we turn each corner along the path only at squares on the existing route. Our path, for example, can go through the Evansville–Cleveland box but cannot turn a corner; thus we cannot place a plus or minus sign there. We may use occupied squares only as stepping-stones:

$$\text{Des Moines–Cleveland index} = \$3 - \$5 + \$8 - \$4 + \$7 - \$5 = +\$4$$

Figure C.6

Testing Des Moines to Cleveland

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	\$5 100 −	\$4	\$3 Start +	100
(E) Evansville	\$8 200 +	\$4 100 −	\$3	300
(F) Fort Lauderdale	\$9	\$7 100 +	\$5 200 −	300
Warehouse requirement	300	200	200	700

Again, opening this route fails to lower our total shipping costs.

Two other routes can be evaluated in a similar fashion:

$$\text{Evansville–Cleveland index} = \$3 - \$4 + \$7 - \$5 = +\$1$$
$$(\text{Closed path} = EC - EB + FB - FC)$$
$$\text{Fort Lauderdale–Albuquerque index} = \$9 - \$7 + \$4 - \$8 = -\$2$$
$$(\text{Closed path} = FA - FB + EB - EA)$$

INSIGHT ▶ Because this last index is negative, we can realize cost savings by using the (currently unused) Fort Lauderdale–Albuquerque route.

LEARNING EXERCISE ▶ What would happen to total cost if Arizona used the shipping route from Des Moines to Cleveland? [Answer: Total cost of the current solution would increase by $400.]

RELATED PROBLEMS ▶ C.1c, C.3c, C.4–C.11 (C.12, C.13 are available in MyOMLab)

EXCEL OM Data File **ModCExC3.xls** can be found in MyOMLab.

In Example C3, we see that a better solution is indeed possible because we can calculate a negative improvement index on one of our unused routes. *Each negative index represents the amount by which total transportation costs could be decreased if one unit was shipped by the source–destination combination*. The next step, then, is to choose that route (unused square) with the *largest* negative improvement index. We can then ship the maximum allowable number of units on that route and reduce the total cost accordingly.

What is the maximum quantity that can be shipped on our new money-saving route? That quantity is found by referring to the closed path of plus signs and minus signs drawn for the route and then selecting the *smallest number found in the squares containing minus signs*. To obtain a new solution, we add this number to all squares on the closed path with plus signs and subtract it from all squares on the path to which we have assigned minus signs.

One iteration of the stepping-stone method is now complete. Again, of course, we must test to see if the solution is optimal or whether we can make any further improvements. We do this by evaluating each unused square, as previously described. Example C4 continues our effort to help Arizona Plumbing arrive at a final solution.

Example C4

IMPROVEMENT INDICES

Arizona Plumbing wants to continue solving the problem.

APPROACH ▶ Use the improvement indices calculated in Example C3. We found in Example C3 that the largest (and only) negative index is on the Fort Lauderdale–Albuquerque route (which is the route depicted in Figure C.7).

SOLUTION ▶ The maximum quantity that may be shipped on the newly opened route, Fort Lauderdale–Albuquerque (FA), is the smallest number found in squares containing minus signs—in this case, 100 units. Why 100 units? Because the total cost decreases by $2 per unit shipped, we know we would like to ship the maximum possible number of units. Previous stepping-stone calculations indicate that each unit shipped over the FA route results in an increase of 1 unit shipped from Evansville (E) to Boston (B) and a decrease of 1 unit in amounts shipped both from F to B (now 100 units) and from E to A (now 200 units). Hence, the maximum we can ship over the FA route is 100 units. This solution results in zero units being shipped from F to B. Now we take the following four steps:

1. Add 100 units (to the zero currently being shipped) on route FA.
2. Subtract 100 from route FB, leaving zero in that square (though still balancing the row total for F).
3. Add 100 to route EB, yielding 200.
4. Finally, subtract 100 from route EA, leaving 100 units shipped.

Note that the new numbers still produce the correct row and column totals as required. The new solution is shown in Figure C.8.

Total shipping cost has been reduced by (100 units) × ($2 saved per unit) = $200 and is now $4,000. This cost figure, of course, can also be derived by multiplying the cost of

Figure **C.7**

Transportation Table: Route FA

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5 / 100	$4	$3	100
(E) Evansville	$8 / 200	$4 / 100	$3	300
(F) Fort Lauderdale	$9	$7 / 100	$5 / 200	300
Warehouse demand	300	200	200	700

STUDENT TIP

FA has a negative index: FA (+9) to FB (−7) to EB (+4) to EA (−8) = −$2

Figure **C.8**

Solution at Next Iteration (Still Not Optimal)

From \ To	(A) Albuquerque	(B) Boston	(C) Cleveland	Factory capacity
(D) Des Moines	$5 / 100	$4	$3	100
(E) Evansville	$8 / 100	$4 / 200	$3	300
(F) Fort Lauderdale	$9 / 100	$7	$5 / 200	300
Warehouse demand	300	200	200	700

shipping each unit by the number of units transported on its respective route, namely: 100($5) + 100($8) + 200($4) + 100($9) + 200($5) = $4,000.

INSIGHT ▶ Looking carefully at Figure C.8, however, you can see that it, too, is not yet optimal. Route EC (Evansville–Cleveland) has a negative cost improvement index of –$1. Closed path = EC − EA + FA − FC.

LEARNING EXERCISE ▶ Find the final solution for this route on your own. [Answer: Programs C.1 and C.2, at the end of this module, provide an Excel OM solution.]

RELATED PROBLEMS ▶ C.4–C.11 (C.12–C.13 are available in MyOMLab)

Special Issues in Modeling

Demand Not Equal to Supply

LO C.3 *Balance* a transportation problem

A common situation in real-world problems is the case in which total demand is not equal to total supply. We can easily handle these so-called unbalanced problems with the solution procedures that we have just discussed by introducing *dummy sources* or *dummy destinations*. If total supply is greater than total demand, we make demand exactly equal the surplus by creating a dummy destination. Conversely, if total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand. Because these units will not in fact be shipped, we assign cost coefficients of zero to each square on the dummy location. In each case, then, the cost is zero.

Degeneracy

LO C.4 *Deal* with a problem that has degeneracy

To apply the stepping-stone method to a transportation problem, we must observe a rule about the number of shipping routes being used: *The number of occupied squares in any solution (initial or later) must be equal to the number of rows in the table plus the number of columns minus 1.* Solutions that do not satisfy this rule are called *degenerate*.

Degeneracy
An occurrence in transportation models in which too few squares or shipping routes are being used, so that tracing a closed path for each unused square becomes impossible.

Degeneracy occurs when too few squares or shipping routes are being used. As a result, it becomes impossible to trace a closed path for one or more unused squares. The Arizona Plumbing problem we just examined was *not* degenerate, as it had 5 assigned routes (3 rows or factories + 3 columns or warehouses − 1).

To handle degenerate problems, we must artificially create an occupied cell: That is, we place a zero or a *very* small amount (representing a fake shipment) in one of the unused squares and *then treat that square as if it were occupied.* The chosen square must be in such a position as to allow all stepping-stone paths to be closed.

Summary

The transportation model, a form of linear programming, is used to help find the least-cost solutions to system-wide shipping problems. The northwest-corner method (which begins in the upper-left corner of the transportation table) or the intuitive lowest-cost method may be used for finding an initial feasible solution. The stepping-stone algorithm is then used for finding optimal solutions. Unbalanced problems are those in which the total demand and total supply are not equal. Degeneracy refers to the case in which the number of rows + the number of columns −1 is not equal to the number of occupied squares. The transportation model approach is one of the four location models described earlier in Chapter 8 and is one of the two aggregate planning models discussed in Chapter 13. Additional solution techniques are presented in Tutorial 4 in MyOMLab.

Key Terms

Transportation modeling (p. 730)
Northwest-corner rule (p. 732)
Intuitive method (p. 733)
Stepping-stone method (p. 734)
Degeneracy (p. 738)

Discussion Questions

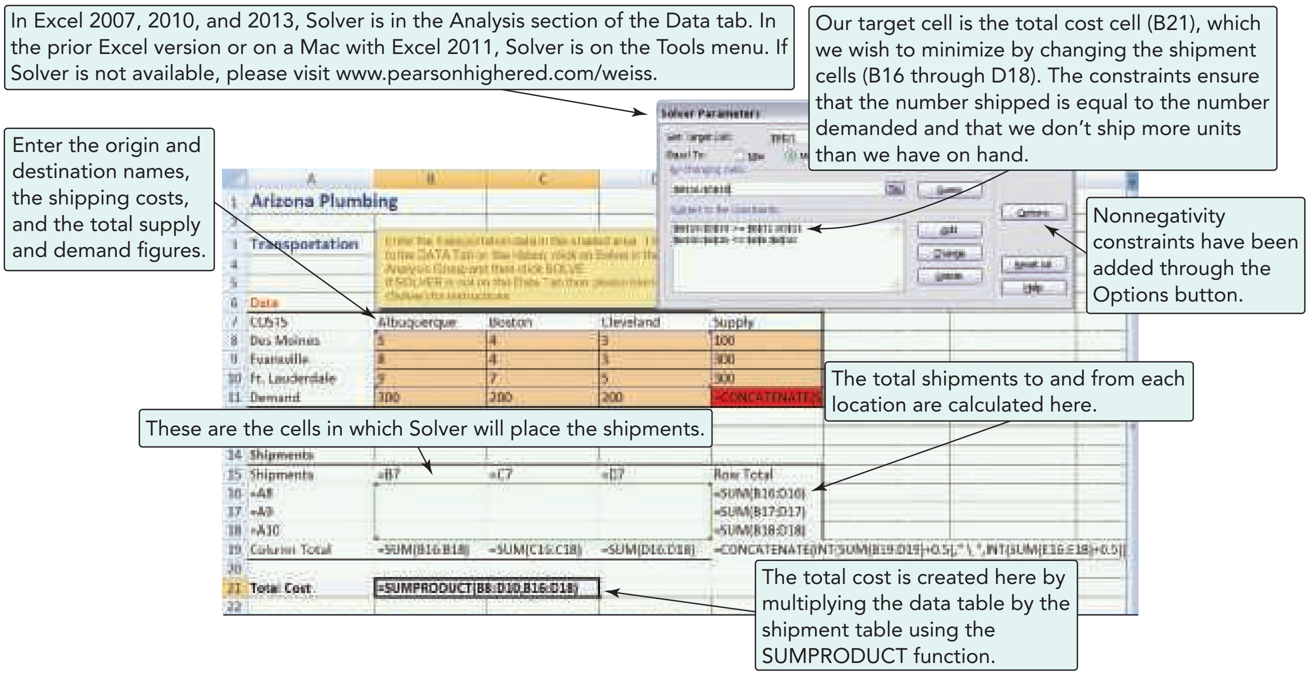
1. What are the three information needs of the transportation model?
2. What are the steps in the intuitive lowest-cost method?
3. Identify the three “steps” in the northwest-corner rule.
4. How do you know when an optimal solution has been reached?
5. Which starting technique generally gives a better initial solution, and why?
6. The more sources and destinations there are for a transportation problem, the smaller the percentage of all cells that will be used in the optimal solution. Explain.
7. All of the transportation examples appear to apply to long distances. Is it possible for the transportation model to apply on a much smaller scale, for example, within the departments of a store or the offices of a building? Discuss.
8. Develop a *northeast*-corner rule and explain how it would work. Set up an initial solution for the Arizona Plumbing problem analyzed in Example C1.
9. What is meant by an unbalanced transportation problem, and how would you balance it?
10. How many occupied cells must all solutions use?
11. Explain the significance of a negative improvement index in a transportation-minimizing problem.
12. How can the transportation method address production costs in addition to transportation costs?
13. Explain what is meant by the term *degeneracy* within the context of transportation modeling.

Using Software to Solve Transportation Problems

Excel, Excel OM, and POM for Windows may all be used to solve transportation problems. Excel uses Solver, which requires that you enter your own constraints. Excel OM also uses Solver but is prestructured so that you need enter only the actual data. POM for Windows similarly requires that only demand data, supply data, and shipping costs be entered.

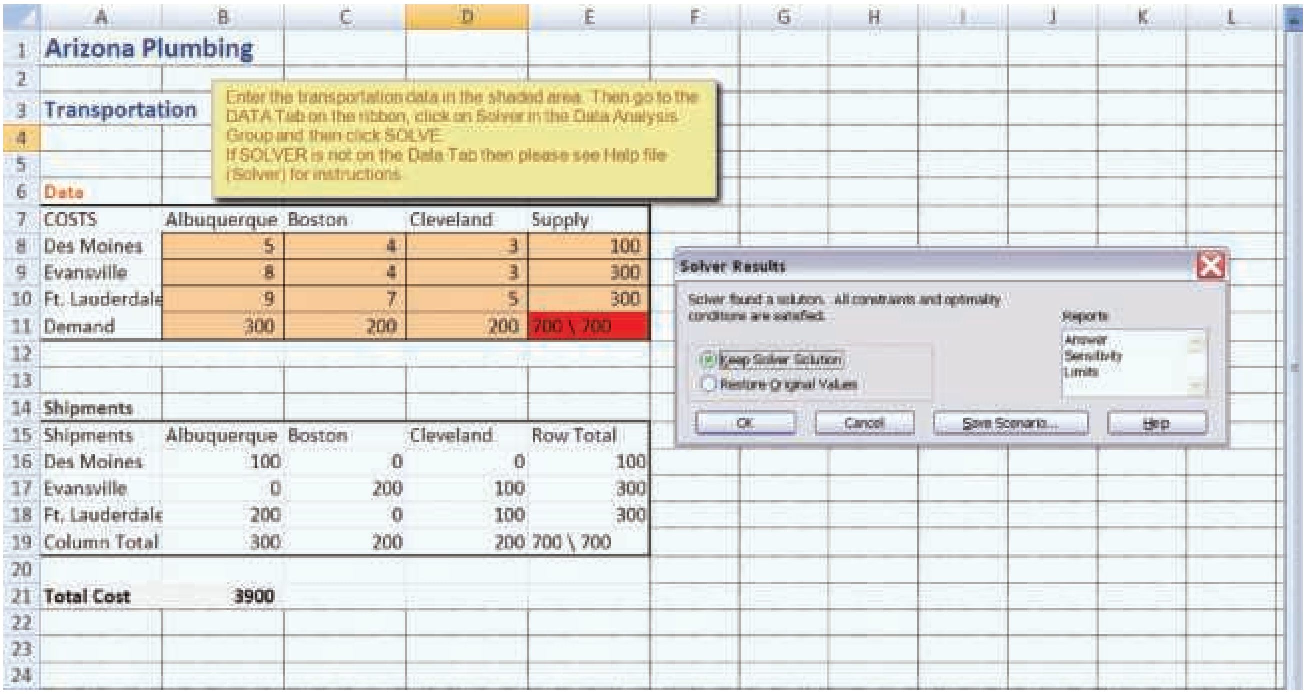
USING EXCEL OM

Excel OM’s Transportation module uses Excel’s built-in Solver routine to find optimal solutions to transportation problems. Program C.1 illustrates the input data (from Arizona Plumbing) and total-cost formulas. In Excel 2007, 2010, and 2013 **Solver** is in the **Analysis** section of the **Data** tab. Be certain that the solving method is “Simplex LP.” The output appears in Program C.2.



Program **C.1**

Excel OM Input Screen and Formulas, Using Arizona Plumbing Data



Program **C.2**

Output from Excel OM with Optimal Solution to Arizona Plumbing Problem

USING POM FOR WINDOWS

The POM for Windows Transportation module can solve both maximization and minimization problems by a variety of methods. Input data are the demand data, supply data, and unit shipping costs. See Appendix IV for further details.

Solved Problems

Virtual Office Hours help is available in MyOMLab.

SOLVED PROBLEM C.1

Williams Auto Top Carriers currently maintains plants in Atlanta and Tulsa to supply auto top carriers to distribution centers in Los Angeles and New York. Because of expanding demand, Williams has decided to open a third plant and has narrowed the choice to one of two cities—New Orleans and Houston. Table C.3 provides pertinent production and distribution costs as well as plant capacities and distribution demands.

Which of the new locations, in combination with the existing plants and distribution centers, yields a lower cost for the firm?

TABLE C.3 Production Costs, Distribution Costs, Plant Capabilities, and Market Demands for Williams Auto Top Carriers

	TO DISTRIBUTION CENTERS			
FROM PLANTS	LOS ANGELES	NEW YORK	NORMAL PRODUCTION	UNIT PRODUCTION COST
Existing plants				
Atlanta	$8	$5	600	$6
Tulsa	$4	$7	900	$5
Proposed locations				
New Orleans	$5	$6	500	$4 (anticipated)
Houston	$4	$6[a]	500	$3 (anticipated)
Forecast demand	800	1,200	2,000	

[a] Indicates distribution cost (shipping, handling, storage) will be $6 per carrier between Houston and New York.

SOLUTION

To answer this question, we must solve two transportation problems, one for each combination. We will recommend the location that yields a lower total cost of distribution and production in combination with the existing system.

We begin by setting up a transportation table that represents the opening of a third plant in New Orleans (see Figure C.9). Then we use the northwest-corner method to find an initial solution. The total cost of this first solution is $23,600. Note that the cost of each individual "plant-to-distribution-center" route is found by adding the distribution costs (in the body of Table C.3) to the respective unit production costs (in the right-hand column of Table C.3). Thus, the total production-plus-shipping cost of one auto top carrier from Atlanta to Los Angeles is $14 ($8 for shipping plus $6 for production).

$$\begin{aligned}\text{Total cost} &= (600 \text{ units} \times \$14) + (200 \text{ units} \times \$9) \\ &\quad + (700 \text{ units} \times \$12) + (500 \text{ units} \times \$10) \\ &= \$8{,}400 + \$1{,}800 + \$8{,}400 + \$5{,}000 \\ &= \$23{,}600\end{aligned}$$

Is this initial solution (in Figure C.9) optimal? We can use the stepping-stone method to test it and compute improvement indices for unused routes:

Improvement index for Atlanta–New York route:

$$\begin{aligned}&= +\$11 \text{ (Atlanta–New York)} - \$14 \text{ (Atlanta–Los Angeles)} \\ &\quad +\$9 \text{ (Tulsa–Los Angeles)} - \$12 \text{ (Tulsa–New York)} \\ &= -\$6\end{aligned}$$

Improvement index for New Orleans–Los Angeles route:

$$\begin{aligned}&= +\$9 \text{ (New Orleans–Los Angeles)} \\ &\quad -\$10 \text{ (New Orleans–New York)} \\ &\quad +\$12 \text{ (Tulsa–New York)} \\ &\quad -\$9 \text{ (Tulsa–Los Angeles)} \\ &= \$2\end{aligned}$$

Because the firm can save $6 for every unit shipped from Atlanta to New York, it will want to improve the initial solution and send as many units as possible (600, in this case) on this currently unused route (see Figure C.10). You may also

From \ To	Los Angeles	New York	Production capacity
Atlanta	$14 — 600	$11	600
Tulsa	$9 — 200	$12 — 700	900
New Orleans	$9	$10 — 500	500
Demand	800	1,200	2,000

Figure C.9

Initial Williams Transportation Table for New Orleans

From \ To	Los Angeles	New York	Production capacity
Atlanta	$14	$11 — 600	600
Tulsa	$9 — 800	$12 — 100	900
New Orleans	$9	$10 — 500	500
Demand	800	1,200	2,000

Figure C.10

Improved Transportation Table for Williams

want to confirm that the total cost is now $20,000, a savings of $3,600 over the initial solution.

Next, we must test the two unused routes to see if their improvement indices are also negative numbers:

Index for Atlanta–Los Angeles:

$$= \$14 - \$11 + \$12 - \$9 = \$6$$

Index for New Orleans–Los Angeles:

$$= \$9 - \$10 + \$12 - \$9 = \$2$$

Because both indices are greater than zero, we have already reached our optimal solution for the New Orleans location. If Williams elects to open the New Orleans plant, the firm's total production and distribution cost will be $20,000.

This analysis, however, provides only half the answer to Williams's problem. The same procedure must still be followed to determine the minimum cost if the new plant is built in Houston. Determining this cost is left as a homework problem. You can help provide complete information and recommend a solution by solving Problem C.7 (on p. 742).

SOLVED PROBLEM C.2

In Solved Problem C.1, we examined the Williams Auto Top Carriers problem by using a transportation table. An alternative approach is to structure the same decision analysis using linear programming (LP), which we explained in detail in Business Analytics Module B.

SOLUTION

Using the data in Figure C.9 (p. 740), we write the objective function and constraints as follows:

$$\text{Minimize total cost} = \$14X_{\text{Atl,LA}} + \$11X_{\text{Atl,NY}} + \$9X_{\text{Tul,LA}} + \$12X_{\text{Tul,NY}} + \$9X_{\text{NO,LA}} + \$10X_{\text{NO,NY}}$$

Subject to:

$$X_{\text{Atl,LA}} + X_{\text{Atl,NY}} \le 600 \quad \text{(production capacity at Atlanta)}$$

$$X_{\text{Tul,LA}} + X_{\text{Tul,NY}} \le 900 \quad \text{(production capacity at Tulsa)}$$

$$X_{\text{NO,LA}} + X_{\text{NO,NY}} \le 500 \quad \text{(production capacity at New Orleans)}$$

$$X_{\text{Atl,LA}} + X_{\text{Tul,LA}} + X_{\text{NO,LA}} \ge 800 \quad \text{(Los Angeles demand constraint)}$$

$$X_{\text{Atl,NY}} + X_{\text{Tul,NY}} + X_{\text{NO,NY}} \ge 1200 \quad \text{(New York demand constraint)}$$

Problems

Note: Px means the problem may be solved with POM for Windows and/or Excel OM.

Problems C.1–C.3 relate to Developing an Initial Solution

• **C.1** Find an initial solution to the following transportation problem.

	TO			
FROM	**LOS ANGELES**	**CALGARY**	**PANAMA CITY**	**SUPPLY**
Mexico City	$ 6	$18	$ 8	100
Detroit	$17	$13	$19	60
Ottawa	$20	$10	$24	40
Demand	50	80	70	

a) Use the northwest-corner method. What is its total cost?
b) Use the intuitive lowest-cost approach. What is its total cost?
c) Using the stepping-stone method, find the optimal solution. Compute the total cost. Px

• **C.2** Consider the transportation table at right. Unit costs for each shipping route are in dollars. What is the total cost of the basic feasible solution that the intuitive lowest-cost method would find for this problem? Px

	Destination					
Source	A	B	C	D	E	Supply
1	12	8	5	10	4	18
2	6	11	3	7	9	14
Demand	6	8	12	4	2	

• **C.3** Refer to the table that follows.
a) Use the northwest-corner method to find an initial feasible solution. What must you do before beginning the solution steps?
b) Use the intuitive lowest-cost approach to find an initial feasible solution. Is this approach better than the northwest-corner method?
c) Find the optimal solution using the stepping-stone method.